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18CS36

Third Semester B.E. Degree Examination, Aug./Sept.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define proposition, tautology, contradiction. Determine whether the following compound statement is a tautology or not.

$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$ **(06 Marks)**

b. Using the laws of logic, show that

$(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ **(07 Marks)**

c. Establish the validity of the following argument :

$\forall x,\ p(x) \vee q(x)$
$\exists x,\ \neg p(x)$
$\forall x,\ \neg q(x) \vee r(x)$
$\underline{\forall x,\ s(x) \rightarrow \neg r(x)}$
$\therefore \exists x,\ \neg s(x)$ **(07 Marks)**

OR

2 a. Define Converse, Inverse and Contrapositive of a conditional. Find converse, inverse and contrapositive of $\forall x, (x > 3) \rightarrow (x^2 > 9)$, where universal set is R. **(06 Marks)**

b. Test the validity of the following arguments:

(i) If there is a strike by students, the exam will be postponed but the exam was not postponed
$\therefore$ there was no strike by students

(ii) If Ram studies, then he will pass in DMS. If Ram doesn't play cricket, then he will study. Ram failed in DMS.
$\therefore$ Ram played cricket **(06 Marks)**

c. Let $p(x) : x \geq 0$
$q(x) : x^2 \geq 0$ and $r(x) : x^2 - 3x - 4 = 0$, then
for the universe completing of all real numbers, find the truth value of
(i) $\exists x \{p(x) \wedge q(x)\}$ (ii) $\forall x \{p(x) \rightarrow q(x)\}$ (iii) $\exists x \{\phi(x) \wedge r(x)\}$ **(06 Marks)**

d. Define dual of logical statement. Write the dual of the statement
$(p \vee T_o) \wedge (q \vee F_o) \vee (r \wedge s \wedge T_o)$ **(02 Marks)**

Module-2

3 a. Define well ordering principle and prove the following by mathematical induction.

(i) $1^2 + 3^2 + 5^2 + \ldots\ldots(2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$

(ii) $1*3 + 2*4 + 3*5 + \ldots\ldots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$ **(12 Marks)**

b. Find the coefficients of
(i) x^9y^3 in the expansion of $(2x - 3y)^{12}$
(ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$ **(08 Marks)**

OR

4 a. Prove that for any positive integer n,
$$\sum_{i=1}^{n}\frac{f_{i-1}}{2^i}=1-\frac{f_{n+2}}{2^n},$$ f_n denote the Fibonacci number. **(06 Marks)**

b. Determine the coefficient of xyz^2 in the expansion of $(2x-y-z)^4$. **(07 Marks)**

c. How many positive integers n, can we form using the digits 3, 4, 4, 5, 5, 6, 7, if we want n to exceed 5,000,000? **(07 Marks)**

Module-3

5 a. If A = {1, 2, 3, 4, 5} and there are 6720 injective functions $f : A \rightarrow B$, what is | B |? **(03 Marks)**

b. Six books each of Physics, Chemistry, Mathematics and four books of Biology totally contains 12225 pages. Find the least number of pages contained in a book. **(05 Marks)**

c. The set A = {1, 3, 4, 7, 9} and B = {2, 4, 6, 7, 8} and $f : R \rightarrow R$ is given by f(x) = 2x + 5. Verify the following results for
(i) $f(A\cup B) = f(A) \cup f(B)$
(ii) $f^{-1}(A\cup B) = f^{-1}(A) \cup f^{-1}(B)$
(iii) $f^{-1}(A\cap B) = f^{-1}(A) \cap f^{-1}(B)$ **(12 Marks)**

OR

6 a. Let A = { 1, 2, 3, 6, 9, 12, 18 } and define R on A by xRy if "x divides y". Draw the Hasse diagram for the poset (A, R). Also write the matrix of relation. **(08 Marks)**

b. Consider poset whose Hasse diagram is given below. Consider B = {3, 4, 5}. Find the upper and lower bounds of B, least upper bound and greatest lower bound of B (Refer Fig.Q6(b)).

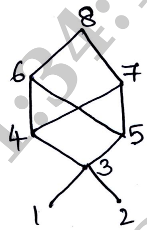

Fig.Q6(b) **(04 Marks)**

c. Let f, g, h : R→R where $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2+2}$. Show that $(h\circ g)\circ f = h\circ(g\circ f)$. **(08 Marks)**

Module-4

7 a. In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(08 Marks)**

b. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that atleast one letter gets to right person. **(04 Marks)**

c. Solve the recurrence relation $a_n - a_{n-1} - 12(n+1)^3, n \geq 1, a_0 = 3$. **(08 Marks)**

OR

8 a. A person invests some amount at the rate of 11% annual compound interest. Determine the period for this principal amount to get doubled. **(06 Marks)**

b. How many permutations of 1, 2, 3, 4, 5, 6, 7, 8 are not dearrangements? **(07 Marks)**

c. Find the rook polynomial for 3×3 board using the expansion formula. **(07 Marks)**

Module-5

9 a. Merge sort the list –1, 7, 4, 11, 5, –8, 15, –3, –2, 6, 10, 3. **(06 Marks)**

b. Determine whether the following graphs are isomorphic or not. [Refer Fig.Q9(b)]

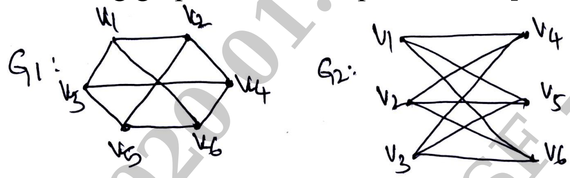

Fig.Q9(b) **(06 Marks)**

c. Define the following with an example to each :
(i) Simple graph (ii) Complete graph (iii) Tree (iv) Regular graph
(v) Spanning subgraph (vi) Induced sub graph (vii) Complete Bipartite graph
(viii) Complement of graph. **(08 Marks)**

OR

10 a. Let G : (V, E) be a connected undirected graph, what is the largest possible value for |V| if |E| = 19 and deg(V) $\geq$ 4 for all v $\in$ V? **(06 Marks)**

b. Construct an optional prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(08 Marks)**

c. T : (V, E) is a complete m-ary tree with | V | = n, if T has ℓ leaves and i internal vertices, prove the following results:
(i) $n = mi + 1$
(ii) $\ell = (m - 1)i + 1$
(iii) $i = \dfrac{\ell - 1}{m - 1} = \dfrac{n - 1}{m}$ **(06 Marks)**

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